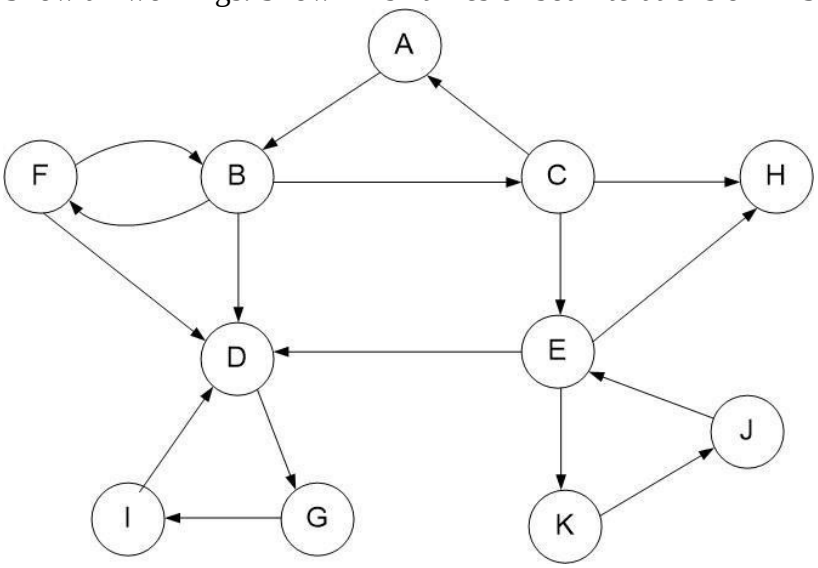
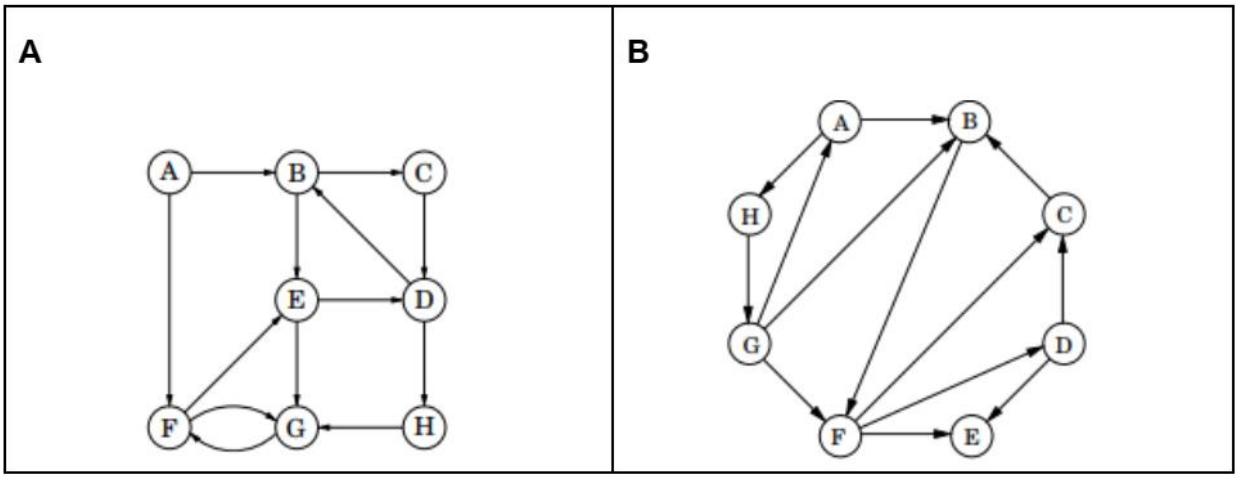
Design and Analysis of Algorithms

Total Marks = 60, each question carries 10 marks.

Q1) Determine the strongly connected components of the graph using the algorithm discussed in class. Show all workings. Show finish times of both iterations of DFS.



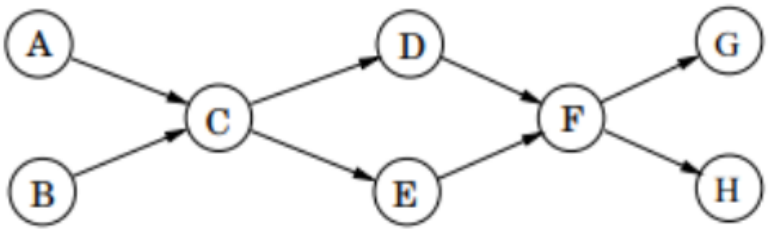
Q2) Run the DFS algorithm on the following graph. Draw the DFS tree (or forest). Label each node with pre and post numbers. Indicate the backward, forward and cross edges. Wherever multiple options are available, visit the nodes in the alphabetic order.



Q3) Run the DFS-based topological ordering algorithm on the following graph. Whenever you have a choice of vertices to explore, always pick the one that is alphabetically first.

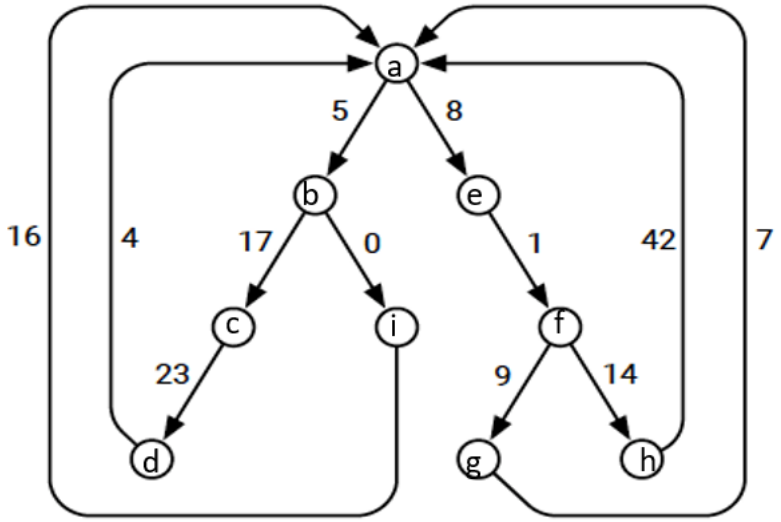
a) Indicate the pre and post numbers of the nodes.
b) What are the sources and sinks of the graph?

c) What topological ordering is found by the algorithm?
d) How many topological orderings does this graph have?



Q4) The reverse of a directed graph G = (V, E) is another directed graph $G^R = (V, E^R)$ on the same vertex set, but with all edges reversed; that is, $E^R = \{(v, u): (u, v) \in E\}$. Give a linear-time algorithm for computing the reverse of a graph in the adjacency list format. Give a clearly specified pseudo-code.

Q5) The following picture shows an example of a Korchoff Graph. It is a weighted, directed graph. As you can see the graph has a binary tree component at is center. Moreover,there are edges going from each leaf of the binary tree backto the root. All edges are weighted and the weights are non-negative. We wish to compute single-source shortest paths in a Korchoff graph.



For example, the shortest path costs from vertex 'b' to all other vertices are as follows:
b -> a: 16, (b, i, a)
b -> c: 17 (b,c)
b -> i: 0 (b,i)
b -> d: 40 (b,c,d)
b -> e: 24, (b, i, a, e)
b -> f: 25, (b, i, a, e, f)
b -> g: 34, (b, i, a, e, f, g)
b -> h: 39, (b, i, a, e, f, h)

a) Is the following statement true or false? Justify your answer:
Dijkstra's shortest path algorithm takes O(|V|lg|V|) time on a Korchoff graph.

b) Design a Single Source shortest path algorithm for the Korchoff graph which works in linear time. The call to the algorithm looks like: spKorchoff(G=(V, E), s, r), where G is a Korchoff graph, s is the source node used to measure the shortest paths, and r is the root node of the binary tree component of the graph. First describe the algorithm in few lines (or steps) in English, then write the pseudocode.

Q6) Agent Bob is living in his houseat A in the enemy territory. There are n other agents in the area, stationed at hotels h_1, h_2, …, h_n. They all must visit Bob's house at A for a top secret meeting. However, the area is constantly monitored and there is a risk of being monitored associated with every road. You have a map, G=(V, E), of the n hotels h_1, h_2, …, h_n and A. This map shows every road between any two hotels or between a hotel and A and also shows its associated risk value, which is a positive weight. The total risk of a path taken by an agent is the sum of the risk values of all the edges in that path. You need to write an algorithm that produces paths for all agents, such that the total risk for each agent is minimized. Your algorithm should work in O((|V|+|E|)lg|V|).